Talks of the conference Categorification in Algebra, Geometry and Physics

A conference in honor of the 60th birthday of Christian Blanchet

Nathan Geer

Re-normalized TQFT's

In this talk I will discuss some new Topological Quantum Field Theories (TQFTs) arising from a re-normalization of the Reshetikhin–Turaev quantum invariants. I will start by giving an axiomatic definition of the re-normalized quantum invariants. These invariants distinguish manifolds that the usual R-T invariants can not and lead to a generalized Volume Conjecture. I will give Atiyah's original definition of a TQFT. Then I will explain how the "'universal construction" of Blanchet, Habegger, Masbaum and Vogel can be applied to the re-normalized invariants and leads to such TQFTs. I will finish the talk by discussing how these new TQFTs lead to mapping class group representations. These mapping class group representations have interesting properties including the fact that the action of certain Dehn twists have infinite order. Such behavior is in sharp contrast with the usual quantum representations of mapping class groups where all the Dehn twists have finite order.

This work is joint with Blanchet, Costantino and Patureau-Mirand.

Sergei Gukov

New structures in knot homologies and invariants of smooth 4-manifolds

In this talk I will discuss new predictions for homological knot invariants and smooth structures on 4-manifolds based on the physical framework that involves a baby version of string theory. String theory is famous for its robust features, such as requirement of 10 space-time dimensions. Likewise, its little cousin discovered in 1995 is rather remarkable and lives in 6 dimensions. When put on a 3-manifold or a 4-manifold, this 6-dimensional theory leads to a theory in the remaining dimensions that knows about delicate invariants of 3-manifold or 4-manifold geometry. Instead of describing the physics of this framework, I will focus on its mathematical outcome that can be put in a form of concrete predictions for knot homology, categorification of quantum groups, and invariants of smooth 4-manifolds.

Kazuo Habiro

Traces and categorification

The trace $\mathrm{Tr}(C)$ of a linear category C is the space of the endomorphisms in C modulo the relations $fg = gf$. It is also known as the 0th Hochschild–Mitchell homology. The trace may be regarded as a decategorification functor, which takes a category (with structure) and gives a set (with structure). For example, the trace of a monoidal linear category naturally has a ring structure. Similarly, the trace of a linear 2-category is a linear category. In some sense, the trace of a linear monoidal category or linear 2-category may be regarded as the space of linear skeins of (2-)morphisms in the annulus. In this talk, I plan to explain some basic general facts about the trace and explain recent results about the traces of the categorified quantum groups.

This talk is based on works joint with Beliakova, Guliyev, Lauda, Webster and Živković.

Joel Kamnitzer

An annular skew Howe duality and K-theoretic quantum geometric Satake

Skew Howe duality connects gl_n with the representation category of SL_m. I will explain an "annular" version of skew Howe duality which connects affine gl_n with the category of SL_m-equivariant coherent sheaves on SL_m. I will then apply this construction to study the equivariant K-theory of Steinberg-like varieties constructed using the affine Grassmannian. In this way, we are able to prove a quantum K-theoretic version of the geometric Satake correspondence.

This is joint work with Sabin Cautis.

Thang Le

On the Chebyshev–Frobenius homomorphism for 3-manifold skein modules at roots of 1

We extend the Kauffman bracket skein modules of 3-manifolds to marked 3-manifolds and show how the the Chebyshev–Frobenius homomorphism appears naturally in this theory.

Anthony Licata

The zigzag algebra, Artin–Tits groups, and Categorification

In studying Weyl groups, it is useful to consider the action of the Weyl group on the roots, and especially the interplay of the Weyl group with the decomposition of roots into positive and negative. All of this structure has a categorified analog, wherein the Weyl group is replaced by the associated Artin–Tits braid group and the root lattice is replaced by the derived category of modules over the zigzag algebra. In this talk we will recall the definition of both the zigzag algebra and the associated representation of the Artin–Tits braid group, and explain how this representation is useful for both studying the group and for other constructions in categorification.

Robert Lipshitz

The Khovanov cube in the Burnside category

We will discuss a refinement of Khovanov homology as a stable equivalence class of 2-functors from the cube to the Burnside category, as well as relationships to previous work of L-Sarkar and Hu-Kriz-Kriz and some (modest) applications.

This is joint work with Tyler Lawson and Sucharit Sarkar.

Ciprian Manolescu

The triangulation conjecture

The triangulation conjecture stated that any n-dimensional topological manifold is homeomorphic to a simplicial complex. It is true in dimensions at most 3, but false in dimension 4 by the work of Casson and Freedman. In this talk I will explain the proof that the conjecture is also false in higher dimensions. This result is based on previous work of Galewski–Stern and Matumoto, who reduced the problem to a question in low dimensions (the existence of elements of order 2 and Rokhlin invariant one in the 3-dimensional homology cobordism group). The low-dimensional question can be answered in the negative using a variant of Floer homology, Pin(2)-equivariant Seiberg–Witten Floer homology. At the end I will also and discuss a related version of Heegaard Floer homology.

Gregor Masbaum

An application of TQFT to modular representation theory

We use TQFT to obtain previously unknown dimension and character formulas for some highest weight modules for symplectic groups over finite fields in the natural characteristic.

Hiraku Nakajima

Towards a mathematical definition of Coulomb branches of 3d $N = 4$ gauge theories

Suppose a pair of a compact Lie group G and its quaternionic representation M is given. Physicists associate a hyper-Kaehler manifold, called the Coulomb branch, to (G, M). I will explain our proposal for a mathematically rigorous definition of its coordinate ring, when M is of a form $N \oplus N^*$, based on the affine Grassmannian of G.

This is a joint work with A. Braverman and M. Finkelberg.

Krzysztof Putyra

The annularization of the Khovanov platform algebras

Given a monoidal 2-category one defines its annularization by considering it as a 3-category with a single object and taking its horizontal trace. For instance, the annularization of a Temperley–Lieb 2-category is a category of points on a circle, flat tangles in an annulus, and cobordisms in a thickened annulus. In my talk I will discuss an annularization of the Khovanov's 2-category of bimodules over platform algebras A^n. This new 2-category produces an invariant of annular tangles, which in the case of links matches the annular Khovanov homology. In addition, I'll discuss a general categorical construction producing Khovanov homology in any thickened surface.

This is a joint work with A. Beliakova.

Hoel Queffelec

Link invariants from a doubled Schur algebra

The HOMFLY-PT polynomial is a two-variable knot invariant, that can be specialized to both the Alexander and the Jones polynomials. However, the quantum groups based constructions yielding these latter invariants do not lift to the HOMFLY-PT polynomial. Using ideas from Howe duality, we introduce a doubled version of the quantum Schur algebra, which allows us to define in a unified quantum setting the HOMFLY-PT, Reshetikhin–Turaev, and Alexander polynomials. This new feature suggests interesting categorification approaches, connecting to joint works with Aaron Lauda and David Rose.

This is a joint with Antonio Sartori.

Peter Samuelson

Double affine Hecke algebras and the torus

The double affine Hecke algebra $H(\mathfrak{g}; q, t)$ of type $\mathfrak{g}$ is a quantum algebra associated to a Lie group G depending on two parameters q, t in C^*. When $q = t = 1$, this algebra is isomorphic to the ring of functions on the G-character variety of the torus (for nice enough $\mathfrak{g}$). We will give evidence that when $t = 1$, $H(\mathfrak{g}; q, 1)$ is isomorphic to the $\mathfrak{g}$-skein algebra of the torus (at least when skein relations of type $\mathfrak{g}$ are defined), and describe a conjectured action on the skein modules of knot complements (for arbitrary q, t and $\mathfrak{g} = \mathfrak{sl}_2$). This raises the question of whether the second parameter t can be "seen by categorification."

Joshua Sussan

A categorical braid group action at a prime root of unity

Khovanov introduced the subject of Hopfological algebra as an approach to categorifying the Witten–Reshetikhin–Turaev 3-manifold invariant. We review the categorification of the upper half of small quantum sl(2) at a prime root of unity due to Khovanov and Qi. Then we explain how this gives rise to a categorification of a braid group action at a prime root of unity.

This is joint work with You Qi.

Pierre Vogel

Functoriality of Khovanov homology

In this paper we prove that every Khovanov homology associated to a Frobenius algebra of rank 2 can be modified in such a way to produce a TQFT on oriented links, that is a monoidal functor from the category of cobordisms of oriented links to the homotopy category of complexes.

Ben Webster

Isomorphisms of knot homologies and skew Howe duality

In recent years, a very powerful framework for finding isomorphisms between different categorifications of type A quantum invariants has been developed by Cautis and others building on his work, using a skew Howe dual presentation of this invariant inside a single irreducible module over sl_∞. I'll discuss this technique, its application in a variety of situations from category O to the Fukaya category, and in particular, recent work of the author and Mackaay directly categorifying skew Howe duality via Koszul duality.

Stephan Martin Wehrli

Annular Khovanov homology and knotted Schur–Weyl representations

Annular Khovanov homology is a homology theory for knots and links in thickened annuli which generalizes Khovanov homology. In this talk, I will show that this homology theory carries an action of the exterior sl(2) current algebra and, for links which are n-cables of knots, a commuting action of the symmetric group S_n. I will also discuss some consequences and generalizations of this result.

This is joint work with Elisenda Grigsby and Anthony Licata.

Geordie Williamson

Representations of symmetric groups, tilting modules and Soergel bimodules

One of the origins of the theory of categorification of Lie algebras lies in the beautiful LLT conjectures relating representations of the symmetric group to affine Lie algebras. I will briefly recall this picture, as well as the theory of tilting modules which provides a bridge between the symmetric group and the general linear group in positive characteristic. Then I will describe a conjecture (joint with Simon Riche) which describes the category of tilting modules in terms of Soergel bimodules for the affine Weyl group.