Counting problems and geometric answers
in non positive curvature

Lectures by
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Circle Gauss' problem, namely to give the number $N(R)$ of integer points within a disk of radius R in the Euclidian plane can be the starting point of lectures on calculus, on number theory (from different standpoints), on geometry of quadratic forms, on harmonic analysis on groups, on dynamical systems in Riemannian manifolds, on geometric group theory,*etc.* This series of lectures is intended to state the corresponding question when considering quotients of Hadamard manifolds X by discrete group of isometries Γ and describe some results. In this non homogeneous framework, the aim is to provide a quantitative description of orbit points of Γ acting on X (for instance the asymptotic of $N(R)$) and a qualitative one (like equidistribution type results). We will put emphasis on probabilistic arguments which turn out to be powerful for this question. We assume the audience to be familiar with hyperbolic and Riemannian geometry but we will recall the essential knowledge if requested. The following preliminary list of references comes as an indication of (some part of) the content of the lectures.

Basics on hyperbolic and riemannian geometry :

Références

[1] Ballmann W., Gromov M., Schroeder V. *Manifolds of non positive curvature*, Progress in Math. **61** Birkhäuser, Boston (1985).

[2] Eberlein P. *Geometry of nonpositively curved manifolds*, Chicago Lectures in Mathematics, University of Chicago Press 1996.

[3] Ratcliffe J.G. *Foundations on Hyperbolic Geometry*, Springer Verlag 1994

[4] Sakai, T. *Riemannian Geometry*, Translations of Mathematical Monographs Vol. 149 1996

More advanced readings on the subject :

Références

[1] Babillot M., Feres R., Zeghib A. *Rigidité, groupe fondamental et dynamique*, Panorama et synthèse. **13** SMF (2002). (See inside the contribution of M. Babillot)

[2] Dal'bo F., Peigné M., Picaud J.C., Sambusetti A. *On the growth of nonuniform lattices in pinched negatively curved manifolds*, J. Reine Angew. Math 627 (2009) .

[3] Knieper G. *The uniqueness of the measure of maximal entropy for geodesic flows on rank 1 manifolds*,- Annals of Math. **148** (1998) p.291-314.

[4] Link G. *Asymptotic geometry and growth of conjugacy classes of Nonpositively Curved Manifolds*, Annals of Global Analysis and Geometry **31** : 37-57 (2007) .

[5] Roblin T. *Ergodicité et équidistribution en courbure négative*, Mémoires S.M.F. n.95 (2003).