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# Local Cohomology: an algebraic introduction with geometric applications

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This book provides a careful and detailed algebraic introduction to Grothendieck's local cohomology theory, and provides many illustrations of applications of the theory in commutative algebra and in the geometry of quasi-affine and quasi-projective varieties. Topics covered include Castelnuovo-Mumford regularity, the Fulton-Hansen connectedness theorem for projective varieties, and connections between local cohomology and both reductions of ideals and sheaf cohomology.

It is designed for graduate students who have some experience of basic commutative algebra and homological algebra, and also for experts in commutative algebra and algebraic geometry.

416 pages, 5 illustrations.

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